Determining payments for watershed services by hydro-economic modeling for optimal water allocation between agricultural and municipal water use

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1. Introduction

Ever growing demand for agricultural and municipal water, caused by population growth and the need to feed the world, as well as increasing stress over water bodies crave for efficient and sustainable water management. Especially in areas where municipal and agricultural water consumption rely on the same water sources for satisfying their water needs, it is important to explore evidence-based policy instruments that achieve sustainable water use in a way that is optimal for both dwellers and farmers concurrently. Some economists regard market-based policy instruments superior to command-and-control instruments in enhancing the economically efficient use of natural resources [1,2]. While some clearly favor market-based instruments in ecosystem management [3,4], such instruments are not the dominant policy strategy for environmental protection [5]. Instead, many authors call for hybrid instruments that combine market-based and command-and-control strategies [5,6]. According to Vatn [7], command-and-control is essential for the functioning of ecosystem markets. Muradian and Rival [5] argue that hybrid regimes that combine command-and-control, market based tools and community-based institutional arrangements are more suitable in managing ecosystem services—which so often raise challenges due to their common good character and intrinsic complexity—than pure markets or hierarchies.

Payments for environmental services (PES) offer incentives to landowners to adopt practices that enhance a certain ecosystem or environmental service (ES) [8]. Wunder [9] defines PES strictly as: “a voluntary transaction in which a well-defined ES (or a land-use likely to secure that service) is being ‘bought’ by a (minimum one) ES buyer from a (minimum one) ES provider if and only if the ES provider secures ES provision (conditionality)” [9]. However, this implies a high level of commoditization of the resource in question and therefore a less strict definition is often applied. According to Vatn [7], most PES schemes are not even incomplete markets. The proper definition of PES has been an ongoing endeavor since Wunder [9,10]. Nevertheless, a clear aim of PES schemes is to encourage individual landowners or land users to adopt practices that are privately unprofitable, but socially desirable, by making those practices more profitable to the individual [1]. When the environmental service in question relates to water, and the landowners are paid to change their land use practices in the expectation of downstream benefits, the term payments for watershed services (PWS) is often used [11]. Hereafter we will use the abbreviation PWS where appropriate.

Watershed processes are highly complex and determining the rate of the payment that ensures the desired outcome for the downstream community is a key issue in designing an efficient PWS scheme [11]. The payments should be set between the seller's opportunity cost and buyer's willingness to pay, but very few (Latin American) PWS schemes actually determine these points [12]. The opportunity cost of land use or land management change could be determined with an auction or by conducting a willingness to pay (WTP) survey [13], or through a set of proxy variables that determine, for instance, the foregone profits, land rent and willingness to accept (WTA) the change [14]. Typically three types of mechanisms are used for price determination: (1) administratively
determined (non-negotiable) payments, (2) direct negotiation between the buyer and the seller, and (3) negotiation through an intermediary [13]. Yet, the design of the negotiations has a large influence on the possibility to attain an efficient solution [15,16]. It is striking that, despite a considerable and steadily growing amount of publications on design of and experiences with PWS schemes, studies so seldom consider the implications of an economically non-optimal payment rate. Admittedly acceptability and social consequences of PWS deserve full attention, but a severely off-mark payment rate can lead to persistence of unsustainable outcomes. The acceptability is not the core of this paper, but we will give it some consideration in the reflective discussion (Section 5.2).

We present a novel theoretical hydro-economic model to determine the payment rate in a payment scheme that allocates water between agricultural and municipal users. Our model contains a specific spatial condition (geographically heterogeneous land quality) and user definition (agricultural and municipal water users as well as a social planner), and is focused on upstream agricultural production and municipal water consumption in downstream areas. Hydro-economic models are typically developed as constrained optimization problems and the objective function includes economic measures of the benefits and costs of water use, whereas hydrological and institutional factors are represented as constraints [17]. The economic concept of opportunity cost determines the demand of water in hydro-economic models by different types of equations, meaning that water demands are not fixed [18]. To our knowledge, no other economic-theoretically underpinned model determining a socially optimal payment rate for watersheds services has been presented in the literature. The model is based on profit maximizing behavior of agricultural and municipal users, who each optimize water withdrawals from the river at given prices and costs. The demand for agricultural water depends on land quality that changes across the agricultural land, production costs and the choice of crop, while the demand for municipal water equals that for potable water. After the analytical framework is developed, we provide a numerical illustration that is applied to the Chancay-Huaral watershed in coastal Peru. The Peruvian coast is identified as a possible future hotspot for water scarcity [19]. The coastline consists of an arid coastal plain and is bordered by the Andean Cordillera. Already under the current climate, intra- and inter-annual water availability in the Chancay-Huaral watershed causes water stress [20,21]. For instance, droughts have led to reorientation of agropastoral economy [21]. Yet, climate change may not exacerbate droughts in the future in this particular area [22].

Assuming that farmers are eventually interested in their income risks caused or heightened by the variability in water availability, the paper also briefly investigates how crop price variability could be accounted for in the proposed PWS approach. This is done with special reference to the extent that crop price variability is related to irrigation water availability. The details of this model extension and the results of crop price volatility impacts on PWS scheme are presented in Appendix 2.

The paper is organized as follows. First, we present the theoretical framework and develop the model itself. This is followed by a numerical application of the model and the results of the numerical solution. The paper ends with a discussion and conclusions.

2. Theoretical framework

2.1. Intuitive reasoning of the model

Two types of participants are distinguished in the payment scheme: agricultural users (farmers) are located upstream and municipal users downstream. They are assumed to follow profit maximizing behavior and in the case of agricultural users it is also assumed that profit equals the farmer's income. Our research hypothesis is that if municipal water consumption is to take precedence in future water allocation in the catchment, the agricultural users may have to restrain their water use in case of low water availability. Such restraint will be more easily exercised if agricultural users are compensated for possible income losses induced by reduced yields. The aim of the PWS scheme is thus to incentivize changes in agricultural land management practices so that downstream water availability increases and the demand for municipal water is fulfilled. In this particular case, the agricultural users are compensated for lost profits due to regulating the used quantity of water. The municipal water users on the other hand are those paying for the secured quantity of water and the embodied loss compensation to agricultural users.

We formulate and solve the model in three stages. In the first stage, we determine the private optimum and formulate separate profit functions for agricultural and municipal water use in absence of PWS, meaning that each actor only pursues its own maximum profit without counting for externalities caused to others [23]. The privately optimal agricultural water application and land allocation are determined by using agricultural prices and water response functions as inputs. For the municipal water supply system, water demand is derived with the point expansion method [24]. In the private optimum, water is open-access commons [23] without regulative measures nor hydrological constraints. Both agricultural and municipal users are able to maximize their profits and satisfy their entire demand. The results of the first stage thus provide the demand for water without regulation or concerns about water availability. The second stage solves the social optimum, meaning that such overall water use levels for both actors are sought which maximize overall profit of both groups, together embodying the local society. In the social optimum, we use the concept of a 'social planner': a neutral actor (for instance, an agency) aiming at optimality for society as a whole. In this case, the social planner allocates water between the two uses (agricultural and municipal) by considering water availability and taking into account the profit maximizing behavior of agricultural and municipal water users. The socially optimal water application, land allocation and amount of potable water are determined by inserting a hydrological constraint to the model. The hydrological constraint represents the maximum available annual water quantity in the catchment available for consumptive uses. If the constraint is binding, limited water availability may actually reduce agricultural yields. In practice, determining the hydrological constraint may be subject to a consultation process with local experts about observed or projected water availability, but, for the purpose of the model presented here, it will remain exogenous. Along with the hydrological constraint we use the concept of shadow price for water reflecting the scarcity of the resource [23]. In the third stage, the PWS scheme is introduced. We use the results of the two previous stages to determine the
payment rate that results in a socially optimal outcome. Subsequently, we compare the private and social optima, introduce a subsidy for agricultural water use and place a tax on municipal water use. Thus, the social optimum is reached through the two instruments, which together constitute the payment scheme.

In addition to water availability, farmers are also subject to other income risks. The price volatility of crop prices is an important source of income uncertainty [25], which can feed back into water demand. This implies that farmers are inclined to overuse water, as insufficient irrigation can reduce yield and unit price in case of notable quality reduction. Crop price volatility has a number of sources, and can be divided in short- and long term volatility [26,27]. Most kinds of price volatility can be tackled by insurance and financial products, as well as crop diversity. However, the volatility that relates to the effect of water availability on yield and subsequently on crop price can be controlled by hydrological management. Therefore, assuming that a farmer wishes to manage income risk, the price volatility of crops can be a relevant addition to the model outlined in Section 2.1; we present this extension in Appendix 2. Especially in the case of low income farming, the regret of income loss has a significant behavioral effect [28] and influences the farmer's attitude (and acceptance) toward PWS. An illustration of how PWS can contribute to the reduction of income volatility can alleviate reservations among farmers toward PWS. This ties in with the alternative uses of the received compensation in the PWS scheme. For instance, investments in improved irrigation efficiency can reduce the vulnerability of the crop yields. We discuss this in Section 5.

2.2. Private optimum

2.2.1. Agriculture

Water demand for irrigation is derived demand that evolves from the value of agricultural products [29]. Agricultural water demand depends on crop mix and timing, water application, and irrigation technology [18]. Especially the type and cost of irrigation technology have a large impact on total water use [30]. For example, the typical irrigation efficiency for surface irrigation systems is between 40 and 80% while for sprinkler irrigation systems it is between 65 and 90% [31]. Crops have varying responses to water, depending on both climatological and agrological factors [32], such as land quality [33]. In addition to land quality, market prices of crops, input costs, water availability, water price, and the risk and management effort (farming practices) that is needed influence the choice of crops [18].

In this modeling exercise, we assume that there are two crops that can be grown and that land allocation to those crops depends on how they manage in different land qualities and how profitable they are in those qualities. The choice of crop and final profit maximizing land allocation are therefore determined by land quality and the profitability of each crop in a particular land quality [33]. Furthermore, agricultural land is assumed to be heterogeneous in quality and divided into parcels, such that each parcel contains land of the same quality level, while the quality differs across the parcels. Hence, each parcel has a different level of optimal water application and optimal yields. It is also assumed that only one crop at a time can be cultivated in each parcel. We also assume that the time scale with respect to water availability and water consumption is one year, regardless of whether some crops have more than one planting cycle per year.

Effective water is the amount of water that is actually utilized by plants and applied water is the amount of water that is collected and applied to the field, whereas irrigation efficiency is the ratio of effective water to applied water. Irrigation efficiency is determined by soil characteristics, for instance water retention capacity and the water application method [34]. Therefore, the amount of applied water differs from the amount that is actually utilized by the crop.

The relationship between agricultural production (y), effective water, applied water and irrigation efficiency is given in equation (1):

\[ y_i = f(e_j) = f(a_{ij}h(q)) \]  

(1)

where \( i \) represents the crop type, \( j \) the parcel, \( e \) effective water, \( a \) the volume of applied water and \( h(q) \) a typical irrigation efficiency of chosen technology with certain soil quality \( q \) (normalized to an index) [29,34,35].

The effective water determines the growth of the crop. Intuitively thinking, the growth of a crop is rather slow in the beginning, but at a certain stage the growth accelerates. When the crop reaches maturity, continuing to apply water at the same pace as previously may be detrimental, and the crop may even be lost. This logic is supported by Caswell and Zilberman [34] and Cai et al. [36] who found that a quadratic form better fits the actual water use pattern of many crops than a Cobb-Douglas function. Therefore, the agricultural production function is assumed to be quadratic with respect to effective water.

\[ f(a_{ij}h(q)) = \alpha_i + \beta_i a_{ij}h(q) - \gamma_i a_{ij}^2 h(q)^2 \]  

(2)

here \( \alpha \), \( \beta \) and \( \gamma \) are parameter values for a certain parcel and crop.

The agricultural profit maximizer makes water use decisions for each of her parcels based on the following profit maximization problem:

\[ \sum_{j=1}^{n} \pi_j = \max_{a_{ij}} \sum_{j=1}^{n} \left[ p_j (\alpha_i + \beta_i a_{ij}h(q) - \gamma_i a_{ij}^2 h(q)^2) - w_i a_{ij} - c_i \right] \]  

(3)

in which \( n \) is the number of parcels, \( p \) price of the output crop, \( w \) unit cost for water and \( c \) the fixed cost for producing the crop.

The agricultural decision maker decides to allocate her land between two different crops. Following Lichtenberg [33], the choice of the profit maximizing land allocation (cropping pattern) is incorporated in the model. The agricultural profit function becomes:
\[ \max \pi_t = \int_0^1 [\pi_t L_t(q) + \pi_0 (1 - L_t(q))] g(q) dq \]

where 1 and 2 stand for crop types and \( L_t(q) \) is a share of land of quality \( q \) used to cultivate crop 1. The land allocation choice \( L_t(q) \) is endogenously determined between the two crops.

Finally, the agricultural problem is to maximize profits with respect to applied water and land allocation:

\[ \max \pi_t = \int_0^1 \left[ \left( p_1 (\alpha_1 + \beta_{1y} a_{1y} h(q) - \gamma_1 a_{1y}^2 h(q)^2) - w_1 a_{1y} - c_1 L_t(q) + p_2 (\alpha_2 + \beta_{2y} a_{2y} h(q) - \gamma_2 a_{2y}^2 h(q)^2) - w_2 a_{2y} - c_2 \right) \right. \\
\left. - L_t(q) \right] g(q) dq \]

The first order conditions for this problem are:

\[ \frac{\partial \pi_t}{\partial a_{1y}} = L_t(q)(p_1 \beta_{1y} h(q) - 2p_1 \gamma_1 h(q)^2 a_{1y} - w_1) = 0 \]

\[ \frac{\partial \pi_t}{\partial a_{2y}} = (1 - L_t(q))(p_2 \beta_{2y} h(q) - 2p_2 \gamma_2 h(q)^2 a_{2y} - w_2) = 0 \]

\[ \frac{\partial \pi_t}{\partial L_t(q)} = m_1 - m_2 = 0 \]

When land quality is normalized from 0 to 1 and follows an even distribution, the breaking point in land quality, or the point where it is profitable to switch from one crop to another crop is the point where the profit curves intersect, \( \pi_1 = \pi_2 \) [37]. Because the crops are different in terms of their characteristics, a solution for optimal land allocation between the two exists.

### 2.2.2. Municipal water use

Municipal water demand depends on, inter alia, population size, sanitary facilities and other water intensive appliances, cooking activities and gardening and other outdoor needs [38]. Also pricing, income, weather and seasonal factors, population and household composition, and non-price consumption controls are identified as having an impact on municipal water demand [39]. In general, improved living standards tend to increase the demand for fresh water services [40]. Capture, storage, treatment and delivery systems require significant inputs for infrastructure from the municipal water service [17].

Drinking water can be priced with constant unit prices or complex tariff systems (increasing or decreasing block rate pricing) [41,42]. For simplicity reasons, we consider constant unit prices in the model. For individual residents, water is a final consumption good and therefore consumer demand theory provides the basis for determining the total demand [17]. Municipal water demand is found to be both relatively inelastic with respect to price changes (dataset combining elasticities reported in different studies, mean elasticity being −0.41, median −0.35 and standard deviation 0.86) and inelastic with respect to income changes (mean elasticity being 0.43, median 0.24 and standard deviation 0.79) [43]. Reported own-price elasticities lie in the same range for developing and developed countries [44].

The profit maximizing municipal water supply system responds to residential water demand, which is driven by consumer utility and affected by the price of potable water. The municipal water demand is considered to be linear and decreasing with price increase, i.e. \( Q(\hat{p}) = n - m \hat{p} \), even though this is admittedly a notable simplification. The Point Expansion method, which is introduced, for instance, in Griffin [24], allows to determine a linear demand function when price elasticity of demand and one point in the demand curve are known.

The profit function for the water supply system is then as follows:

\[ \max \pi_m = \hat{p} Q(\hat{p}) - \hat{w} Q(\hat{p}) - \hat{c} Q(\hat{p}) - \phi \]

where \( Q(\hat{p}) \) is the derived consumer demand for potable water and thus also for the abstracted water amount. The sales price for potable water set by the water supply system operator is \( \hat{p} \), whereas \( \hat{w} \) is the unit cost for raw water abstraction from the river, \( \hat{c} \) denotes the operating cost of delivering water to end-users, and \( \phi \) is the fixed cost for water purifying activities and other relevant processes. The necessary condition for the municipal water supply system is

\[ \frac{\partial \pi_m}{\partial Q(\hat{p})} = 0 \]

The condition implies that the municipal water supply system withdraws water from the river to the point where the long run marginal costs of providing water equal the price of potable water it receives from the customers. In fact, municipal water supply is often regulated by law, which implies that it may not comply with profit maximizing behavior. In this case, the model could be modified so that it would consider for instance monopolistic behavior or have a different pricing scheme instead of constant unit price. For instance, a block rate pricing where the lowest block rate price would consider the legally binding obligations to provide water to municipal users. We acknowledge the relevance of frequently applied price regulation in this sector. Yet, even if pricing of drinking water is not based on the marginal revenue rule, it is not essentially altering the functionality of the PWS scheme regarding the restraint incentive to farmers, provided that the municipal water consumption is small compared to the water consumption for
irrigation. We discuss the issue further in section 5.2.

2.3. Social optimum

Let us assume that a social planner wishes to increase the water amount allocated to the municipal water supply system. The increase in water for the municipal water supply system is transferred via $M$: $Q(\hat{p})^* = n - m\hat{p} + M$.

Under water scarcity there is a need to allocate water wisely between the two major user categories. Water scarcity refers to a situation where there is not enough water to satisfy all private demand. Hence a hydrological constraint is introduced to create a limit to total water use. From economic efficiency and social welfare maximization perspective, water should be allocated between the two user groups so that their marginal products would again equal the marginal costs, but now the marginal costs include a shadow price $\lambda$ for water scarcity. The objective function for the socially optimal water allocation combines the agricultural and municipal profit functions:

$$W = \int_0^1 [L_1(q)p_1 + (1 - L_1(q))p_2]g(q)\,dq + \pi_M$$

(11)

Subject to

$$A - \sum a_j - Q(\hat{p}) = 0$$

(12)

where $A$ equals the total amount of available water.

Now we have a Lagrangian optimization problem:

$$L = \int_0^1 [(p_1f(a_j h(q)) - w_1a_j - c_1]L_1(q) + [p_2f(a_j h(q)) - w_2a_j - c_1](1 - L_1(q))g(q)\,dq + \hat{p} Q(\hat{p}) - \hat{\omega}Q(\hat{p}) - \hat{\phi} Q(\hat{p}) - \phi$$

$$+ \lambda \left( A - \int_0^1 [a_j L_1(q) + a_j (1 - L_1(q))]g(q)\,dq - Q(\hat{p}) \right)$$

(13)

The first order conditions for this equation are similar to the one for the private optimum with $\lambda$ added to reflect the shadow price of the water availability limit. In this social planner decision framework, the vantage point differs fundamentally, as the allocation of water is governed by the authorities and not by individual user entities, as is the case in the private optimum.

$$\frac{\partial L}{\partial a_j} = L_1(q)(p_1\beta_1 h(q) - 2p_1\gamma_1 h(q)^2 a_j - w_1) - \lambda L_1(q) = 0$$

(14)

$$\frac{\partial L}{\partial a_2} = (1 - L_1(q))(p_2\beta_2 h(q) - 2p_2\gamma_2 h(q)^2 a_2 - w_2) - \lambda (1 - L_1(q)) = 0$$

(15)

$$\frac{\partial L}{\partial L_1(q)} = \pi_1 - \pi_2 = 0$$

(16)

$$\frac{\partial \pi_M}{\partial Q(\hat{p})} = \hat{\pi} - \hat{\omega} - \hat{\phi} - \hat{\lambda} = 0$$

(17)

$$\frac{\partial L}{\partial \lambda} = A - \int_0^1 [a_j L_1(q) + a_j (1 - L_1(q))]g(q)\,dq - Q(\hat{p}) = 0$$

(18)

The shadow price $\lambda$ thus internalizes the externality that too abundant water use upstream can cause for the downstream users in an unregulated water use context (private optimum). The municipality equalizes the water end-user price with the new marginal cost that includes the shadow price of water and withdraws the new equilibrium amount of water. Total water withdrawals must not exceed water availability, but not all water necessarily has to be withdrawn. These are the expected conditions for the model.

When $A$ is known, $\lambda$ can be determined by completing a grid search by calibrating values for $\lambda$ so that the hydrological constraint is satisfied.

2.4. PWS scheme

The PWS scheme could promote the adoption of water conservation technology in different ways: by sharing fixed costs of technology adoption, by subsidizing the reduction in water use, or by combining the two mechanisms [45]. In order to reach the socially optimal level of agricultural water use in this PWS scheme, a subsidy on the reduction of private agricultural water use is offered, so that agriculture is encouraged to decrease water withdrawals to the socially desired level. The volumetric subsidy denotes a payment per decreased unit of water to agricultural irrigation water users. On the other hand, a volumetric tax is levied on municipal water withdrawals to represent the service buyer’s payment. These instruments are chosen to be used in the payment scheme because of the upstream-downstream locations of the two participants and the local setting, and to retain public budget.
neutrality for this measure. Thus, the payment received by agriculture is determined by an input reduction:

$$\max_{\text{subsidy}} = p f (a_{ij}^* h(q)) - w_i a_{ij}^* + s (a_{ij}^* - a_{ij}^t) - c_i$$  \hspace{1cm} (19)$$

Where $a_{ij}^*$ is the socially optimal amount of water, $a_{ij}^t$ is the privately optimal amount of water and $s$ refers to the subsidy. For a subsidy, the first order condition is:

$$\pi_{\text{subsidy}} = p_j h(q) - 2p_j h(q) a_{ij}^* - w_i - s = 0$$  \hspace{1cm} (20)$$

Solving for $s$ the equation yields:

$$s = p_j h(q) - 2p_j h(q) a_{ij}^* - w_i$$  \hspace{1cm} (21)$$

The first order condition with respect to the socially optimal applied water imply that the payment or subsidy should be equal to the marginal revenue.

For the municipal water supply system, the payment is determined as a tax $t$ that is imposed on its water use.

$$\rho_m = \hat{p} Q(\hat{p}) - (\hat{w} + t) Q(\hat{p}) - \hat{c} Q(\hat{p}) - \phi$$  \hspace{1cm} (22)$$

By taking the first order conditions in terms of $Q(\hat{p})$ yields

$$\pi_{\rho_m} = \hat{p} \hat{w} - t - \hat{c} = 0$$  \hspace{1cm} (23)$$

and results in a tax that is determined as:

$$t = \hat{p} \hat{w} - \hat{c}$$  \hspace{1cm} (24)$$

Finally, equations (21) and (24) determine the payment rates for the PWS scheme. The optimal agricultural water use subsidy is calculated by inserting the socially optimal water use values $a_{ij}^*$ into equation (21). The optimal municipal water use tax is calculated by inserting the socially optimal water price into equation (24).

2.5. Crop price volatility

In the preceding sections the prices of crops were given. Alternatively, they can be redefined to allow for water related variability, which introduces a price volatility component. Here we briefly discuss how the crop price volatility could be incorporated in the basic model that was presented in Sections 2.2–2.4, whereas in Appendix 2 we discuss how the price volatility may impact the PWS scheme. In Appendix 2 we also show the crop price volatility effects on farmer profits when only one crop (maize) is cultivated.

We conjecture that the crop price has an exogenous base level applicable under good yield conditions and a correction part that depends on the extent to which available irrigation water falls below a reference level for the entire region or country. A reduced crop production at the national level triggers crop price increases. The price elasticity with respect to aggregate yield variation varies significantly across crops. We refer to this effect on the crop price as the ‘production volume effect’. Dispersed localized shortfalls in water availability are assumed not to lead to production volume effects on the national crop price, while large scale widespread shortfalls (amounting to a national shortfall) do lead to such price effects. In addition to the production volume effect, we introduce also a ‘quality effect’ for crop price volatility. Water shortage, especially more serious irrigation restrictions can easily cause reductions in the average quality of the produce, which as such creates pressures for discounts. We refrain for the moment from other sources of price volatility, as these are more random and exogenous from the point of view of one farm or one farming region [26,27].

In summary, the crop price is redefined as follows:

$$p_i = p_i^* \left(1 + \varepsilon \left(\frac{q - q_i}{q}\right)\right)p_i$$  \hspace{1cm} (25)$$

Where $p_i^*$ denotes the price under good harvest conditions, $\varepsilon$ is the price elasticity with respect to aggregate (national) supply of crop $i$ in response to water availability, $q$ denotes the average maximum attainable harvest for crop $i$ per ha given optimal water use per ha ($\bar{a}$) in the country, $q_i$ is the actual (physical) harvest per hectare for crop $i$ given actual water use $a_i$, and $p_i$ the quality correction factor related to the amount of actual available water per hectare ($a_{ij}$) as compared to the ideal amount $\bar{a}$. Quality discounts on crop prices show a large spread in elasticity in relation to quality features, depending, for instance, on crop, area, kind of quality decrease, availability of quality references [46–48]. In this case, so as to avoid non-essential complications and apply some degree of caution we assume a moderated quality effect on the price:

$$\varphi_i = \sqrt{a_{ij}/\bar{a}}$$  \hspace{1cm} (26)$$

When accounting for water induced crop price volatility, as specified in equation (25), equation (3) is extended as follows:

$$\max_{a_{ij}} \sum_{i=1}^{n} \left[p_i^* \left(1 + \varepsilon \left(\frac{q - q_i}{q}\right)\right)p_i \bar{a} + \beta_{ij} a_{ij} h(q) - \chi a_{ij}^2 h(q)^2 - w_i a_{ij} - c_i\right]$$  \hspace{1cm} (27)$$

We refrain from presenting the numerical solution to this problem in this article but discuss it in Appendix 2.
3. Application of the model

The PWS-model has been developed in the context of the Peru-AquaFutura project, which aims to develop a decision-support system for the use of scarce water resources in Peru. The numerical simulation of the model is conducted with a dataset from the Huaral district in the Chancay-Huaral basin, which is located in the coastal area of Peru, near the capital region of Lima. The Huaral district is the major population center in the basin with approximately 89,000 inhabitants [49]. The Chancay-Huaral basin reaches from the coastal lowlands to the mountains and therefore the climatology in the basin has a strong gradient: the average annual temperature ranges from 2 to 17°C and the average annual accumulated precipitation from 280 to 1000 mm [22].

The hydrological supervision and management of the irrigation area is carried out by a local branch (ALA) of the National Water Authority of Peru (ANA), which inter alia projects the available annual amounts of water for irrigation and levies land owners a charge per hectare of irrigated land, of which the revenues are reinvested in local hydrological management. In other words, agricultural water use is priced, but the price has no link to overall water availability, neither to a customer's actual consumption. On the other hand, decisions on the maintenance of the irrigation system and cropping choices are made by the council of users. More details about the local hydrological management can be found in Refs. [20,22].

The two crops used in the numerical simulation are cotton and maize, since they are very important crops in the area. The data and methods used for preparing the data for the numerical example are presented in Sections 3.1 and 3.2 below. The uncertainties related to the data are also discussed, which should also be taken into account when interpreting the results.

3.1. Data

3.1.1. Municipal water use

Household level data would be valuable in estimating municipal water demand, but the seemingly broad literature on municipal water demand modeling is actually reliant on rather few unique datasets. This is problematic, and is partly explained by the lack of data in the field [39]. Household level water use or income data is not available for the study area either, but a dataset from the municipal water company EMAPA Huaral (Table 1) is utilized in the numerical example.

Table 1 shows the produced volume of municipal water, the daily production per capita, the average water tariff, the operative costs, the share of unbilled water, and the volume of billed water in Huaral 2006–2012. The table shows that a substantial amount of water remains unbilled even though there is a diminishing trend. From the viewpoint of model calibration, this seems rather problematic, since the data does not tell whether the average cost is from the total produced volume or only from the billed volume.

3.1.2. Yields and prices for maize and cotton

The annual average yield per hectare, average price data and average gross revenue per hectare for the two most important crops in the Chancay-Huaral basin in 2000–2011 are shown in Table 2. These data are calculated from the dataset provided by MINAGRI–SIEA [51]. Table 2 shows that the average yield of maize per hectare has increased with time, while the yield of cotton has remained rather steady. The increase in average maize yield may be the result of, for instance, improved farming practices, increased water availability or increased use of fertilizers. Regrettably there is no data about fertilizer use from the area of Chancay-Huaral, but World Bank data of fertilizer consumption in Peru indicates that there has been a slightly increasing trend in fertilizer consumption (kilograms per hectare of arable land) during 2002–2013 [52], although that does not seem to be an explanatory reason for the increased yield of maize.

3.2. Determining the parameters

For maize, the United Nations Food and Agriculture Organization (FAO) indicates a water requirement of 500–800 mm for the whole growing period [53], which corresponds to approximately 5000–8000 m³ per hectare. Let's assume that a yield of 8.8 ton/ha (highest average harvest in Huaral during 2000–2011; see Table 2) is attained by effective water amount that equals 8000 m³ of water and yield of 5.5 ton/ha (smallest average harvest in Huaral during 2000–2011; see Table 2) is attained by effective water amount of 5000 m³ of water.

Parameters α, β, γ for the response function are determined by a system of equations [34]:

Table 1

<table>
<thead>
<tr>
<th>Year</th>
<th>Produced volume (m³)</th>
<th>Production per capita (liters/person/day)</th>
<th>Average tariff (PEN/m³)</th>
<th>Operative cost (PEN/m³)</th>
<th>Unbilled water (%)</th>
<th>Billed water (m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>n/a</td>
<td>269</td>
<td>0.86</td>
<td>1.06</td>
<td>51.4%</td>
<td>n/a</td>
</tr>
<tr>
<td>2007</td>
<td>5,855,885</td>
<td>256</td>
<td>0.89</td>
<td>0.98</td>
<td>48.8%</td>
<td>2,999,608</td>
</tr>
<tr>
<td>2008</td>
<td>6,003,508</td>
<td>251</td>
<td>0.93</td>
<td>1.01</td>
<td>49.1%</td>
<td>3,054,184</td>
</tr>
<tr>
<td>2009</td>
<td>5,929,381</td>
<td>241.3</td>
<td>1.04</td>
<td>0.97</td>
<td>46.1%</td>
<td>3,198,692</td>
</tr>
<tr>
<td>2010</td>
<td>5,813,003</td>
<td>240.2</td>
<td>1.37</td>
<td>1.07</td>
<td>45.4%</td>
<td>3,171,413</td>
</tr>
<tr>
<td>2011</td>
<td>5,896,402</td>
<td>227.2</td>
<td>1.63</td>
<td>1.29</td>
<td>44.9%</td>
<td>3,246,436</td>
</tr>
<tr>
<td>2012</td>
<td>5,960,309</td>
<td>214.8</td>
<td>1.68</td>
<td>n/a</td>
<td>42.8%</td>
<td>3,411,529</td>
</tr>
</tbody>
</table>
This system yields the following water response function for maize:

\[ y = e^{14666.6667 + 5.86e^{0.00036e^{2}}} \]

The same procedure is applied for cotton. The highest average yield during 2000–2011 was 3.2 tons/ha attained in Huaral in 2004 and lowest 2.3 tons/ha in 2000 and 2005. According to FAO, the water requirement is typically between 700 and 1300 mm per growing period \[53\], meaning a total amount of 7000–13000 m$^3$ per hectare. These assumptions yield the following water response function for cotton:

\[ y = e^{1025 + 0.65e^{0.000025e^{2}}} \]

Table 2

<table>
<thead>
<tr>
<th>Year</th>
<th>Maize (Tons/ha)</th>
<th>Average price (PEN/kg)</th>
<th>Gross revenue (PEN/ha)</th>
<th>Cotton (Tons/ha)</th>
<th>Average price (PEN/kg)</th>
<th>Gross revenue (PEN/ha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>5.5</td>
<td>0.6</td>
<td>3300</td>
<td>1.8</td>
<td>0.6</td>
<td>4140</td>
</tr>
<tr>
<td>2001</td>
<td>6.3</td>
<td>0.6</td>
<td>3780</td>
<td>2.5</td>
<td>0.6</td>
<td>5750</td>
</tr>
<tr>
<td>2002</td>
<td>6.7</td>
<td>0.6</td>
<td>4020</td>
<td>2.8</td>
<td>0.6</td>
<td>5040</td>
</tr>
<tr>
<td>2003</td>
<td>6.6</td>
<td>0.6</td>
<td>3960</td>
<td>3.0</td>
<td>0.6</td>
<td>6900</td>
</tr>
<tr>
<td>2004</td>
<td>7.0</td>
<td>0.6</td>
<td>4200</td>
<td>3.2</td>
<td>0.6</td>
<td>8320</td>
</tr>
<tr>
<td>2005</td>
<td>7.7</td>
<td>0.5</td>
<td>3850</td>
<td>3.3</td>
<td>0.6</td>
<td>4830</td>
</tr>
<tr>
<td>2006</td>
<td>8.0</td>
<td>0.5</td>
<td>4000</td>
<td>2.8</td>
<td>0.6</td>
<td>5880</td>
</tr>
<tr>
<td>2007</td>
<td>7.9</td>
<td>0.6</td>
<td>4740</td>
<td>2.6</td>
<td>0.6</td>
<td>7280</td>
</tr>
<tr>
<td>2008</td>
<td>8.4</td>
<td>0.7</td>
<td>5880</td>
<td>2.8</td>
<td>0.6</td>
<td>7000</td>
</tr>
<tr>
<td>2009</td>
<td>8.8</td>
<td>0.3</td>
<td>2640</td>
<td>3.0</td>
<td>0.6</td>
<td>4200</td>
</tr>
<tr>
<td>2010</td>
<td>7.8</td>
<td>0.4</td>
<td>3120</td>
<td>2.7</td>
<td>0.6</td>
<td>7560</td>
</tr>
<tr>
<td>2011</td>
<td>8.2</td>
<td>1.0</td>
<td>6200</td>
<td>2.8</td>
<td>0.6</td>
<td>8120</td>
</tr>
</tbody>
</table>

In order to represent the variation in land quality despite the absence of relevant data, it is decided that the land can be partitioned into 10 different quality parcels. These parcels are assumed to be of equal size, but their land quality differs. Thus, for each parcel, the optimal water application varies. The variation in land quality is endogenously incorporated in the crop response functions by changing parameter $\beta$ which shifts the parable vertically. The production functions are calibrated so that the lowest quality parcel yields the minimum and highest quality parcel yields the maximum yield. Hence the value for $\beta$ changes evenly between 5.435 and 5.687 for maize and 0.578–0.65 for cotton (for maize the change is 0.048 and for cotton it is 0.08 compared to the previous value).

As discussed above, the demand curve for municipal water in Huaral is determined with the point expansion method. This is a simple approach for characterizing municipal water demand when data is lacking \[18\], but for this specific case study area it is considered to be sufficient. For parameterization of the demand function, the own-price elasticity of demand and the price and quantity in one point of the demand curve are required \[24\]. The point in the demand curve is obtained from local data, while the elasticity is established on the basis of various studies focusing on water demand elasticity. With the combination of elasticity of $-0.41$ \[43\], water price 1.63 and water amount 3246436 (2011 data, see Table 1) the demand curve $Q(\hat{p}) = -816588\hat{p} + 4577475$ is established. It should be noted that only water that has been billed is accounted for in the calibration of the municipal water demand. Since there is no data for fixed costs of the EMAPA Huaral, it is left out from the numerical example. However, it is assumed that the municipal water supply system follows a zero-profit condition. According to the data, operational costs are lower than the price of drinking water. To correct this, we add another constant to the operational costs so that $\hat{p} = \hat{c}$.

The last adjustment required for the simulation is related to the units of measurement for water. Agricultural water in Chancay-Huaral basin is billed according to hectares irrigated. In Huaral the price is on average PEN 114.34 (Peruvian Sol) per hectare \[54\], which equals approximately EUR 30 per hectare. However, the model incorporates water volumes in cubic meters which implies that an approximate value per cubic meter of agricultural water should be determined. For a hectare of cotton, the water requirement is at most 13000 m$^3$ and for a hectare of maize 8000 m$^3$. Hence, it is assumed that a farmer considers that the approximate price per cubic meter of water is the hectare based cost divided by assumed water use. This yields that the price per cubic meter of water used for cotton is PEN 0.0088 and for maize PEN 0.0143.

Since one of the paper’s main aims is to illustrate the utility of PWS in managing scarce water resources, in this numerical example, water availability $A$ is chosen to be lower than total private water demand. The value for $A$ is set to 350 000 000 m$^3$. Due to lack of information from the site, we set the hydrological constraint arbitrarily; however, this does not change the conclusions that one can draw from the model. The shift of drinking water demand curve in the social optimum is chosen arbitrarily at 1 000 000. All the parameter values are summarized in Appendix 1.
4. Results

The model is solved in stages with Excel.

The results of water use and profits under the three modeling stages (private optimum, social optimum, and the PWS scheme) are shown in Table 3 and Table 4. With the PWS scheme, agricultural water use decreases and municipal water use increases to the socially optimal level. The social optimum [equations (14)–(18)] is obtained when \( \lambda \) is 0.05. Therefore, the payment from the municipal water supply system operator to the agricultural users should be PEN 0.05 per saved cubic meter of water. For cotton parcels this equals PEN 107.8 per hectare and for maize PEN 21.3 per hectare. It should be noted, however, that it would be highly recommendable when water charges would be levied by m\(^3\) water deviated from the river, and thereby also a uniform PWS rate per saved m\(^3\) could be applied. Otherwise, a hectare based water charge would be complemented with a PWS rate that incites water intensive crop choices.

The agricultural profits in the PWS scheme are higher than in the private optimum, which implies that the agricultural water users would be likely to accept the PWS scheme. The municipal profits comply in all cases with the zero-profit condition.

In order to be applicable in practice, the proposed PWS scheme should be accepted by the consumers of drinking water, since the price for drinking water will increase because the consumers will eventually be the payers. If consumers have the willingness to pay more in exchange for more durable water management, the scheme could be a viable option for allocating water in the area. In the presented numerical example, the drinking water price would increase by 3.1\%, which seems rather acceptable (Table 5).

Table 6 shows for different levels of water availability the PWS water saving subsidy, the drinking water price, and the consequently land allocation by crop. The subsidy increases when the amount of available water decreases. When water gets very scarce, the optimal land allocation changes to favor the less water needy crop, which in this case is maize. In correspondence with growing funding needs for the subsidy the price for drinking water increases when water becomes scarcer.

As explained in section 2.5 the PWS rate \( s \) (and the shadow price \( \lambda \)) are sensitive to crop price volatility. In this respect, it could be considered to define the PWS rate \( s \) in reference to a long-term average level of water availability that is in force several years and is subject to review only once in several years (see Appendix 2). This ties in with recommendation of adequate monitoring of the river discharge, water consumption for irrigation, and price variations of key crops. In Appendix 2 we further discuss the implications of alternative ways to account for crop price volatility in PWS schemes. From the point of view of income security (which will significantly affect farmers’ judgement of the acceptability of PWS schemes) it seems justified to pay at least some attention to interaction effects with crop price volatility.

5. Discussion

5.1. Limitations of the model

In reality, total water abstraction in the Huaral district accounts for about 151 million cubic meters (MCM), of which 145 MCM accounts for agriculture and 6 MCM for municipal water use. The simulated private demand for water in Huaral is 378 MCM, implying that the model greatly overestimates agricultural water demand. Regarding the numerical application of the model (Section 3), we have identified a few issues relating to water demand modeling that may limit the use of the approach in its presented form and would thus be excellent future research topics. As a general note, issues with water demand modeling are always coupled with the availability of data from the site. For instance, the demand modeling would benefit from data about agricultural water demand elasticity with respect to water price and also other data that we discuss below.

The probable reason for the large difference between the model and reality lies behind the calibrations of agricultural demand. First, since actual crop-specific water use data and water requirement data from the area are unavailable, the starting point for the calibration is rather fuzzy. Second, determining the land quality by changing values for \( \beta \) results in yields that do correspond to the historical data, but in all likelihood, it overestimates water use. Third, the model takes into account irrigation efficiency, which in the

<table>
<thead>
<tr>
<th>Total water use</th>
<th>Agricultural water use</th>
<th>Municipal water use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private optimum</td>
<td>377,889,515</td>
<td>374,643,079</td>
</tr>
<tr>
<td>Social optimum</td>
<td>341,063,934</td>
<td>336,858,327</td>
</tr>
<tr>
<td>PWS-scheme</td>
<td>341,063,934</td>
<td>336,858,327</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total profits</th>
<th>Agricultural profits</th>
<th>Municipal profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private optimum</td>
<td>59,820,109</td>
<td>59,820,109</td>
</tr>
<tr>
<td>Social optimum</td>
<td>59,232,102</td>
<td>59,232,102</td>
</tr>
<tr>
<td>PWS scheme</td>
<td>60,330,986</td>
<td>60,330,986</td>
</tr>
</tbody>
</table>
area is only approximately 40% [20]. Interestingly, by assuming an irrigation efficiency of 100%, the simulated private agricultural demand is 154MCM, which is rather close to the reported agricultural demand. The model assumes that 60% of applied water does not reach the plants, meaning that there is a great loss of water. A substantial part of the applied irrigation water is in fact recaptured, both through soil hydrology (infiltration) and technical processes [20,22], implying that a part of the water is applied to the fields more than once. In fact, improvements in the irrigation efficiency of blocks served by more upstream abstraction canals would reduce the water availability risks of blocks served by the most downstream-located abstraction canals. Fourth, the model assumes that only two crops are grown in the area. These two crops may not represent aggregate water demand in the best possible way. Cotton has a high water requirement, which results in high agricultural water demand in the model. Even though cotton is an important crop in the Chancay-Huaral watershed, it does not account for 80% of the land – or even half of it in Huaral district, as the privately optimal solution implies. Altogether, overestimating the agricultural water demand in the model might result in too high PWS payments, which might also affect the scheme acceptability.

Another limitation relates to the fact that the presented model assumes linear demand for potable water and this simplification might also lead to underestimating water demand. On one hand, it has been pointed out that the literature on residential water demand tends to use a linear form regardless of its inconsistency with economic theory and utility maximization [38]. Still, lesser data requirements in determining the linear demand curve with the point expansion method support the simplified approach for determining potable water demand in this case study. Regardless, developing the model so that the demand for potable water is non-linear would better represent true demand, and therefore it is a valid next step. Furthermore, future studies could introduce factors such as population and household composition, living standards and recreational habits to the municipal water demand model, since they do impact municipal water demand [38–40]. This would enable the consideration of broader social processes, such as urbanization and other developmental aspects in PWS scheme and model development.

Lastly, our model considers only annual water allocation and not monthly or seasonal use and supply. Since a part of the crops fit for the area can be planted more than once per year, whereas precipitation occurs usually only in the period January–April, information and price-based guidance should have higher temporal resolution than annual, for instance, being based on quarterly projections covering several quarters ahead. To date, the stochasticity of inflows is addressed only in a few hydro-economic models due to the substantial computation burden it creates [18]. For this reason it is left out from the model presented in this paper.

### 5.2. Implications of the results for sustainable water allocation

Despite the aforementioned limitations in demand modeling, we believe that our approach has potential in determining economically efficient payment rates for watershed services. Obviously the approach needs further validation, but this exercise shows that even with incomplete data an initial PWS assessment can be made. Learning processes can improve the estimates over time. The strength of the proposed approach is in the process rather than in the numerical application. Nevertheless, considering the results of the numerical application, agricultural water users might accept a PWS scheme to compensate for water use limitations, provided that the PWS scheme does not discourage other risk management options and does not lead to growing disparity in wealth among farmers. Therefore, the key factor in the functionality of the scheme is the acceptability of the surcharge on municipal water and of the transfer of the revenues to farmers. If municipal water users are willing and able to pay more for their drinking water, the scheme could be viable both socially and economically. With the presented numerical example, the drinking water price increase in the PWS scheme is 3.1% compared to the price in the private optimum, in which we used the actual price and demand for 2011. This seems quite a reasonable price increase, and is actually the increase that happened 2011–2012 (Table 1). The average water tariffs in Peru have increased slightly in recent years [55]. However, the fact that still over 40% of delivered water has remained hitherto unbilled does cast some doubt about the acceptability of even modest price rises for the municipal water consumers [50]. The acceptability of the PWS scheme for the municipal water users may also be enhanced by the way the agricultural waters users would allocate the

### Table 5

<table>
<thead>
<tr>
<th>Drking water price (PEN)</th>
<th>Change in drinking water price (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private optimum</td>
<td>1.63</td>
</tr>
<tr>
<td>Social optimum</td>
<td>1.68</td>
</tr>
<tr>
<td>PWS scheme</td>
<td>1.68</td>
</tr>
</tbody>
</table>

### Table 6

<table>
<thead>
<tr>
<th>Water availability (A)</th>
<th>PWS rate ($s$) in PEN per m³ water</th>
<th>Land allocation (%)</th>
<th>Drinking water price ($p^*$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Cotton</td>
<td>Maize</td>
</tr>
<tr>
<td>350000000</td>
<td>0.05</td>
<td>80</td>
<td>20</td>
</tr>
<tr>
<td>300000000</td>
<td>0.11</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>250000000</td>
<td>0.18</td>
<td>10</td>
<td>90</td>
</tr>
</tbody>
</table>

revenues of the compensation. If not all revenues are paid out to farmers, but instead a part is used for improvement of the irrigation efficiency, some employment may be created for the urban residents. Furthermore, if the crop price volatility is to be taken into account in the PWS scheme, the price for municipal water might increase more, depending on the degree of responsiveness of the PWS system (see Appendix 2). As crop price volatility will vary between crops, but also over time (years) a somewhat longer term perspective in PWS level definition may be called for, which in due course requires some minimum level of monitoring of the river discharge, water use for irrigation and crop prices. Sophisticated approaches to account for crop price volatility may, however, have limited relevance (see Appendix 2), and probably raises management cost of the PWS system. The dilemmas regarding acceptability and adequate scaling of the PWS scheme to farmers are related to the multi-objective character of the instrument aiming both at more optimal water use and at income stability.

As it is a human right to access water to fulfill the basic needs and this right should be, and often is, protected by law [56], a PWS scheme is only pursuable in situations where municipal users can afford it and it is socially acceptable. In fact, municipal water use restrictions may be politically more attractive to implement than price increases, since increasing municipal water price hits low income households harder than high income households, but water use restrictions induce similar responses regardless of income [57]. The legal obligation to provide water may mean that the municipal water supply system delivers water to the consumers at a lower price than would be profitable in competitive market terms, which is actually the case also in Peru [55] It may also mean that the municipal water supply system applies block rate pricing, which we have not considered in the model. Looking at the PWS scheme from the municipal water supply perspective, non-price (e.g. specific technology adoption) conservation policies decreases total revenues whereas price-based conservation policies to reduce demand increases the revenues [58], which implies that the municipal water supply system would probably approve the scheme, if it can increase the price on the consumers’ expense.

In general, farmers tend to apply too much water and use water less productively when trying to maximize yields [59]. The price of water is the main factor influencing agricultural water use [35], so farmers are responsive to prices in their irrigation water demand. However, when water prices are already extremely high, further price rises constitute an ineffective and socially harmful approach for cutting water demand [60]. Instead of increasing agricultural water price, the proposed PWS model subsidizes the demand cuts. From a theoretical point of view and considering the behavior of agricultural users, the scheme would thus seem potent and relevant.

Many authors call for more attention to the institutional aspects, socio-economic context, social relations, and bargaining power when designing incentive schemes such as PWS [5,14]. The payment model presented in this paper is developed within the context of the current decision structures regarding total water allocation, water use and cropping in the study area, even though the water pricing mechanisms get modified due to PWS. Considering the Peruvian context, local actors are in practice responsible for executing water management (see Section 3). Actual implementation of a PWS scheme would largely depend on these actors. Furthermore, the local municipal water suppliers are also important parties to be consulted. Also, the urban municipal councils in the area could function as representatives of the urban water users (predominantly private households).

We delimited our analysis only to consumptive uses of water and thus have not considered uses that are more likely to affect the quality rather than quantity of water, such as hydropower generation and mining. Yet we acknowledge that, not the least in Peru, water quality is certainly an issue. In principle, the presented set-up of the PWS model (its modeling stages) could be applied to other water sector dilemmas with potentially competing demands, if the demand functions can be adequately fitted to the considered study area. Since a good part of the Peruvian coast consists of an arid coastal plain bordered by the Andean Cordillera, and as this coastline area is a possible future hotspot for water scarcity [19], there are many watersheds similar to Chancay-Huara to which a similar PWS scheme could be applied.

The presented model casts the entire irrigation area as one decision maker, which, in a way, emulates the behavior of the local council of users. For a smooth functioning of the PWS scheme, also good hydrological monitoring (at least at irrigation block level) and modeling is important to support good initial cropping plans, which is important to achieve relatively stable PWS schemes. Considering the current situation, this implies investment in measurement systems, of which the cost should be balanced against the benefits of the PWS scheme. Only after the installation of a water use monitoring system prices can be based on water consumption, thereby getting a more effective control and guidance function on water allocations. Peru is already moving towards measuring the use of water blocks for agricultural irrigation [61]. Booker et al. [17] state that approaches that require volumetric measurements of water use should be considered carefully since they might create perverse incentives or be impossible under existing institutions. The accuracy of the proposed PWS scheme would highly benefit from an adequate monitoring system of water use, but desired reductions in upstream agricultural water use may also be inferred by monitoring the cropping pattern.

5.3. Contribution to the PWS literature

Typically water allocations are not the result of economic optimization [17], but of ad-hoc policy interventions. This is especially true for municipal water use, since it may be regulated by law [56]. A great deal of recent literature has been examining institutional and conceptual aspects [12,62–64] and the performance of existing PWS schemes [62,64,65], but there is barely any literature on payment rates and how they are specified. Yet, it is important to know the economically efficient rate of payment in order to avoid situations where a PWS scheme worsens water allocation. Our approach and the modeling stages has attempted to address this knowledge gap. It should be also noted that this paper highlights the social optimum for water use, which is a slightly different concept than that of a socially acceptable water allocation. Although past literature has found that the terms and conditions of scheme participation [62], institutional aspects relating to fairness [65] and use of in-kind payments [64] contribute to the success of PWS schemes, none has examined the payment rate and its impact on achieving the desired environmental outcome; this study makes
a first step toward this direction.

Our key contribution to the PWS literature is the way we have modelled the PWS scheme. The PWS scheme is a combination of a subsidy imposed on restraint in agricultural water use and a tax imposed on municipal water use. The tax and subsidy are determined by defining private and socially optimal water use levels, and then socially optimal level is reached with these two instruments. We have not found similar approaches in the literature. Regarding the PWS scheme, we believe that the presented approach and modeling stages are sufficient to provide evidence-based guidance for achieving socially and economically sustainable water management. The final applicability of the presented approach would then depend on the used demand functions and availability of data in the considered area. The approach could also be modified to fit other water sector setups than agricultural and municipal water use.

6. Conclusions

This paper presents a model that determines the payment rate in a PWS scheme between agriculture and municipal water supply systems and is a contribution to the scarce literature on the determination of economically efficient payment rates for watershed services. The model consists of three elements: an agricultural model, a municipal water model and a hydrological constraint. The agricultural model is a heterogeneous land quality model that is based on the profit maximizing behavior of an agricultural decision maker. The municipal water supply system operator is also a profit maximizer and its behavior is modelled accordingly, even though the zero-profit condition is applied to its decision making. The hydrological constraint creates a balance between water supply and demand: the total demand that consists of agricultural and municipal demand cannot outweigh the supply. These assumptions create the framework in which the PWS model operates.

Ever increasing demand for water sources locally and the growing stress over water management practices in general crave for new solutions to water allocation issues. Following the line of thought of our research hypothesis, the PWS scheme seems to achieve the desired change in water use in the context of upstream agriculture and downstream municipal water use. The proposed modeling approach also enables the determination of the payment rate in such settings, and is the first attempt in developing a decision support tool to determine the payment rates that will yield socially optimal water allocation. In our PWS scheme model, the volumetric subsidy denotes a payment per decreased unit of water to agricultural irrigation water users and a volumetric tax is levied on municipal water withdrawals to represent the service buyer’s payment. In the numerical application, the municipal water price increase due to PWS is rather modest, which supports the social acceptability of PWS scheme in Chancay-Huaral watershed.

While some state that a marginalist approach for acknowledging the services of nature has its limits in what it can change or achieve [5], we believe that an economically qualified analysis on how to determine the payment rate, rather than setting the payment arbitrarily, is still needed to inform decision-making. Sophisticated hydro-economic models could provide a basis for at least the reallocation of water volumes, if not quality control, and the model presented in this paper is an attempt in that direction. Furthermore, the proposed model approach may be applied to other areas where sufficient data exists and the upstream-downstream setting is similar. The model can be modified to fit other contexts as well, by changing the profit functions according to the sector in question. Future research and model development should focus on improving the methods for water demand modeling and looking at the impact of payment rate in PWS scheme effectiveness.

Conflicts of interest

None.

Acknowledgements

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Appendix 1

Model parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Municipal water demand elasticity with respect to price</td>
<td>n/a</td>
<td>−0.41</td>
<td>PEN/m³</td>
<td>[43]</td>
</tr>
<tr>
<td>Municipal water price</td>
<td>̂p</td>
<td>1.63</td>
<td>PEN/m³</td>
<td>[50]</td>
</tr>
<tr>
<td>Operative cost</td>
<td>c</td>
<td>1.29</td>
<td>PEN/m³</td>
<td>[50]</td>
</tr>
<tr>
<td>Municipal water demand, intercept parameter</td>
<td>n</td>
<td>4,577,475</td>
<td></td>
<td>Model calibration</td>
</tr>
<tr>
<td>Municipal water demand, slope parameter</td>
<td>m</td>
<td>−816588</td>
<td></td>
<td>Model calibration</td>
</tr>
<tr>
<td>Shift in municipal demand</td>
<td>M</td>
<td>1,000,000</td>
<td>n/a</td>
<td></td>
</tr>
<tr>
<td>Price of maize</td>
<td>̂p₁</td>
<td>1.0</td>
<td>PEN/kg</td>
<td>[51]</td>
</tr>
</tbody>
</table>
Crop price volatility impact on the PWS scheme.

The farmers participating in the PWS scheme are concerned about sustaining or increasing their income level. Crop price volatility may influence the farmers’ profits and hence the optimal PWS rate. It may also impact the land allocation of crops. In Section 2.5 we introduced a ‘production volume effect’ and a ‘quality effect’ on crop price. Water availability shortfalls may impact crop prices via both these effects, but the effects vary across crops. For example, large scale water availability shortfalls may affect the crop price via the production volume effect if the shortfall decreases the agricultural output nationwide and via the quality effect if the shortfall has also impact on the quality of produce locally.

If decision making and optimization takes place at the level of individual farms, the production expression following $\epsilon_i$ contains exogenous (fixed) data (in equation (27)). Yet, if cropping decisions are optimized at larger (regional or multi-regional) scale, $q_i$ becomes endogenous and the entire optimization becomes mathematically (and socially) more complex. Therefore, we assume here that water availability induced effects on production volume via $q^*$ (the average maximum attainable harvest) and $q_i$ (the actual (physical) harvest) are exogenous and denoted as $Q$. Based on equations (26) and (27), this results in the following first order condition for irrigation water:

$$\frac{\delta F}{\delta a_{ij}} = 0 = \frac{0.5a_i a_j a_i^{0.5} h(q) - 2.5a_i a_j a_i^{0.5} h(q)^2 + 1.5a_i a_j Q a_i^{0.5} h(q) - 2.5a_i a_j Q a_i^{0.5} h(q)^2}{\sqrt{a_i a_j}} - w$$

The actual amount of water consumption at the aggregate level can only be established ex post, though up-to-date measurement and monitoring systems can shorten the lag. With adequate hydrological and crop modeling projections for likely intervals of expected water demand can be generated. We assume farmers have access to projections of next season’s water availability and access to or knowledge of typical supply price responses to expected and realized crop production.

We approach the crop price volatility impact on the PWS scheme by anticipating four different future scenarios:

Scenario 1: hydrological projections do not indicate significant shortage locally nor nationally. Scenario 2: hydrological projections do indicate significant shortage locally, but not nationally, Scenario 3: hydrological projections do not indicate significant shortage locally, but do so nationally, Scenario 4: hydrological projections indicate significant shortage both locally and nationally.

Table A1 shows the implications of the scenarios for the PWS scheme application. In case of scenarios 1 or 3 the model presented in Sections 2.2–2.4 can be applied, but with few cautions. In case of scenarios 2 and 4, the production volume effect and quality effect may have impacts on the PWS scheme. In scenario 2 it may have impact if the quality effect on crop price of water shortfall is relevant locally and in scenario 4 the impact on the PWS scheme depends on the objectives by which the PWS scheme is steered.

Table A1
The functionality of the PWS system under different local and national water availability conditions.

<table>
<thead>
<tr>
<th>No significant water shortage expected nationally</th>
<th>Significant water shortage expected nationally</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No significant water shortage expected locally</strong></td>
<td><strong>Scenario 1:</strong> Water induced crop price volatility no issue; i.e. $a_i = a_j = a_{ij}$</td>
</tr>
<tr>
<td><strong>Optimal water and crop allocation as in Sections 2.2–2.4</strong></td>
<td><strong>Scenario 3:</strong> Optimal water and crop allocation as in Sections 2.2–2.4</td>
</tr>
<tr>
<td><strong>Other sources of price volatility (e.g. export markets) may still merit attention.</strong></td>
<td><strong>Check whether relevant crop prices are expected to rise significantly; consider adaptation of cropping plan given agro-technical and hydrological boundary conditions; optimal PWS rate (as input reduction</strong></td>
</tr>
</tbody>
</table>
Figs. A1 and A2 show the impact on revenue per hectare for maize at different water availability conditions and different supply price elasticity levels ($\epsilon_i$ in equations (25)–(28)), while applying equation (27) in absence of a PWS scheme. Other crops may have curves with different slopes, which means that the optimal allocation of land by crop will vary according to expected hydrological conditions and crop selection. Crops with high elasticities can appreciably cushion effects of (wide spread) low water availability as compared to situations with only a quality effect. Applied parameter values in Fig. A1 are as in Annex 1 for maize.

The above considerations also apply to the establishment of the PWS rate ($s$). In case both the quality effect and the production volume effect are to be taken into account, the first order condition for optimal PWS rate is written as:

$$s = 0.5p_i \alpha_i a_i^{0.5} + 1.5p_i \beta_i \sigma_i h(q) - 2.5p_i \chi_i a_i^{1.5} h(q)^2 + 1.5q_i \rho_i Q_{ij}^{0.5} h(q) - 2.5q_i \chi_i Q_{ij}^{1.5} h(q)^2 - w_i$$

(28a)
In case only the quality effect is taken into account, the first order condition for the optimal PWS rate collapses to:

\[ s = \frac{0.5p_{a}q_{a}^{0.5} + 1.5p_{b}q_{b}^{0.5}h(q) - 2.5p_{c}q_{c}^{1.5}h(q)^{2}}{\sqrt{w_i}} \]  

(28b)

Fig. A3 shows how the optimal PWS rate (**s**) for maize would vary depending on:

1. Whether the PWS scheme is linked to short term variations in water availability (**Q_{var}**) or to a long term average ideal (**Q_{100}**) or fairly ideal (**Q_{90}**) water situation, where 100 and 90 refer to 100% and 90% of the average maximum water application;

2. The inclusion of the quality effect only (**Q_{var}**, **Q_{100}**, **Q_{90}**) or also price volatility induced by production volume effects while assuming different elasticity levels (0.7; 1.0; 1.5) [**Q_{var}; \varepsilon = 0.7; Q_{100}; \varepsilon = 1; Q_{90}; \varepsilon = 1.5**] [**Q_{100}; \varepsilon = 0.7; Q_{100}; \varepsilon = 1; Q_{100}; \varepsilon = 1.5**] [**Q_{90}; \varepsilon = 0.7; Q_{90}; \varepsilon = 1; Q_{90}; \varepsilon = 1.5**]

As regards the cases where only the quality effect counts, the options **Q_{90}** and **Q_{100}** show a constant PWS level as the payment level is pegged to a long term ideal (**Q_{100}**) or near ideal (**Q_{90}**) water level, and therefore do not react to quality discount effects. The option **Q_{var}** represents the case where the PWS is linked to short term market conditions (driven by water scarcity induced quality shortfall). All these cases reflect scenario 2 from table A1, i.e. only local drought.

If both the quality effect and the production shortfall induced price effect are relevant, the resulting PWS levels for a long term pegged reference water level [**Q_{100}; \varepsilon = 0.7; Q_{100}; \varepsilon = 1; Q_{90}; \varepsilon = 1.5**] [**Q_{90}; \varepsilon = 0.7; Q_{90}; \varepsilon = 1; Q_{90}; \varepsilon = 1.5**] divert significantly from PWS levels based on short term market conditions [**Q_{var}; \varepsilon = 0.7; Q_{var}; \varepsilon = 1; Q_{var}; \varepsilon = 1.5**]. Even though the PWS levels differ substantially the overall costs of the PWS system do not grow to the same extent, as the total amount paid depends on actual water use. In theory a reward for non-used water (as compared to a reference level) could be a purer incentive, but is very hard to operate in practice as measurement of non-used water is very hard to implement reliably. Further control calculations of total income effects with different water restraint situations and differently sensitized PWS schemes (as illustrated above) indicate that water availability is – under the given assumptions – the dominant structural driver of income trends (downward with less water). Yet, the crop price volatility with respect to national crop production levels can cause significant deviations from the trend line. Hence the advice would be to use longer term pegged PWS levels, while assuming a reasonable crop price elasticity, preferably based on (recent) historic evidence.

Long term fixed PWS rate levels provide predictability from a PWS funding point of view, but may get inadequate in case of projected serious droughts. Yet, admittedly PWS is not primarily an income support instrument. On the other hand, PWS schemes responsive to price volatility may lead to high compensation levels in situations where negative income effects for farms are moderate or even absent. All in all, in the current set-up PWS systems do not face only acceptability challenges due to the mere rise of drinking water prices, but also due to wealth transfers while some recipients are anyhow wealthier than some of the payers. Probably, a rather simple PWS system is to be preferred, but during its preparation thorough assessment of induced effects is called for.

Fig. A3. Variation in optimal PWS rate (**s**) under different water availability and supply price elasticity regimes; Results shown for worst soil quality land; high soil quality land entails only slightly higher payment rates.

References


